Scheduling the fixtures for the African Cup of Nations Football Tournament: A "no seeding" approach

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Abstract—In this study, we designed an alternative schedule, which excludes seeding, for the African Cup of Nations (AFCON) Football Tournament. Assuming all other constraints and factors are as arranged by AFCON tournament organizers, a simple random sampling without replacement technique was used to randomly allocate the 16 qualified teams to $n = 4$ preliminary groups: A, B, C, D; 4 teams per group. The concept of symmetric Latin square design was used to schedule the teams' preliminary stage group matches. The quarter-final, semi-final and final stage matches were scheduled using tournament designs. In the resulting schedule which excludes seeding, the teams have equal probability of being assigned to any of the preliminary groups in the tournament. This is an advantage over the schedule being used by AFCON presently.

Index Terms— *Seeding*, *Simple random sampling without replacement*, *Symmetric Latin square*, *Tournament Designs*, *Tournament scheduling*.

1 INTRODUCTION

Tournament scheduling has become an important area of research in recent years. According to [1], professional sporting activities have become a lucrative business and the quality of the schedules greatly determines the amount of revenue accruable to sport organizations. The design of a schedule is determined by many factors. This includes facility availability, time, number of entries and other special constraints. These factors contribute to the level of complexity involved in the construction of the schedules. Different methods, which include combinatorial, constraint and integer programming and so on are used for constructing tournament schedules. Some of these methods can be found in [2], [3], [4], [5], [6], [7], [8]. The major types of tournament scheduling, including round-robin and single elimination tournament scheduling types, are discussed in [9].

Tournament scheduling using combinatorial designs have received a lot of attention in literature. Orthogonal Latin squares were applied to the scheduling of golf tournaments in [10]-[12]. Orthogonal Latin squares were also applied in [13] to the scheduling of a round robin for mixed doubles table tennis (MDTT) tournament of order n where a team of n men and n women opposes another team of n men and n women, equivalent to two orthogonal Latin squares. Constructions on self-orthogonal Latin squares with symmetric orthogonal mates (SOLSSOMs) were used to provide schedules for spouse avoiding mixed double round robin (SAMDRR) tournaments of order $4 \le n \le 20$ in [14]. Latin square was used to schedule a SAMDRR tennis tournament of order $n \neq 2, 3, 6$ in [15]. A whist tournament was scheduled by [16] using a resolvable balanced incomplete block design RBIBD. A motor speedway tournament for 4 riders competing in a 20 heat competition was scheduled by [17] using a balanced incomplete block design. Balanced tournament designs (BTD) for all orders were constructed by [18] using recursive constructions and some other techniques from combinatorial design theory. Graph theory was used to design some tournament schedules in [19].

Canonical patterns on a round robin tournament for $2n$ teams were constructed by [20].

Seeding is an important aspect of most tournament schedules. According to [21], tournaments can either be seeded or randomly drawn. Seeding ensures that stronger teams face weaker teams early in the competition thereby maximizing the probability that the strongest teams will survive to the end of the tournament. This leads to a very interesting final match and huge financial gains for the organizers of the tournament [22]. But this is a major disadvantage for the so called "weaker teams" because they are not given equal opportunity like their "stronger teams" counterparts to display their relative strengths. On the other hand, in random draws, all teams have equal probability of being allocated to groups for the games. This implies that two highly rated teams can meet early in the competition. It has been shown by [23] that expected effort and win-probabilities in any two-player contest do not rest on the absolute strength (win valuations) of the respective players alone, but also on their relative strengths. Thus, this may lead to more effort and thrill that can produce better performance in the so-called "weaker teams". Lately, this has come into play as various cases of "upsets" have occurred in international and local tournaments, thereby discouraging the usage of seeding.

In the AFCON tournament presently, seeding is used in the allocation of the 16 qualified teams to preliminary groups. Therefore, the main objective of this study is to design an al-

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ternative schedule, which excludes seeding, for the AFCON tournament that allocates all 16 qualified teams randomly to preliminary groups and then apply the concept of symmetric Latin square design to schedule the teams' preliminary group matches. Also, we will schedule the quarter-final, semi-final and final stage matches by using the Single elimination format coupled with Berger Tables, a round robin algorithm. The constraints, as provided by the organizers, which governed the 2013 AFCON tournament included the following:

- Participating Teams: 16
- Dates: 19 January 2013 to 10 February 2013
- Number of matches: 32 Matches
  - 24 Group Stages (6 matches in each group)
  - 8 Knock-out matches
- Match Days: 17 match days
- Match Sites: 24 match sites
  - 2 Matches per day for 16 matches (Group Stages) in same venue in the same city. The remaining 8 matches will be played 2 per day but not necessarily the same city.
- Host Cities / Stadiums: 5 Cities, 5 stadiums
- Training Sites
  - 1 per team (4 per Host City)
  - 1 for Referees

This present work is however only concerned with the method for scheduling the fixtures, and assumes that all constraints on organizing a successful competition are satisfied.

This paper is divided into five sections. Section 1 contains the introduction while sections 2 and 3 contain the description of the AFCON football tournament and round robin tournaments. The methodology is presented in section 4 and the conclusion is in section 5.

2 AFRICAN CUP OF NATIONS (AFCON) FOOTBALL TOURNAMENT

This tournament is organized by the Confederation of African Football (CAF) and it is held biennially. Sixteen teams, including the host nation, participate in the tournament. The tournament has two stages: the preliminary and the single elimination stages. The teams are seeded into four groups A, B, C, and D, each containing four teams. According to the rules that governed the 2013 edition of the tournament, for instance, the host nation is assigned the first slot in group A while the title holder from the previous edition of the tournament is assigned the first slot in group C. The remaining 14 teams are ranked based on their performance in the last three editions of the tournament (2008, 2010 and 2012 editions) and points are awarded. Then, random draws are made based on the groupings of the points to determine the particular group for each of the teams. The top two teams from each group advance to the knockout stage. In this stage, the winner of each group plays against the runner-up of another group. This is followed by the semi-finals, the third-place match (contested by the losing semi-finalists), and the final. The details can be found in [24].

3 ROUND ROBIN TOURNAMENTS

A round robin tournament, according to [25] is defined as follows:

> Given the set $Z_{2n} = \{0,1,\ldots,2n-1\}$ (the elements of which are called teams), a round-robin tournament of order $2n$ is a partition into $2n-1$ parts (called rounds) of Z_{2n}, each consisting of 2-subsets (called matches) so that each unordered pair in $Z_{2n}^{(2)}$ (the set of all 2-subsets of Z_{2n}) occurs in exactly one part.

This implies that if n is the number of teams, a pure round robin tournament requires $n(n-1)/2$ games. If n is even, then, in each of $(n-1)$ rounds, it is possible to play $n/2$ games involving all the n teams. If n is odd, there will be n rounds, each with $(n-1)/2$ games, and each team would have no game in a particular round.

Generally, a round robin tournament is a tournament where all teams meet all other teams a fixed number of times. The fixed number of times may be single, double, triple and quadruple. The AFCON tournament is a combination of round robin and single elimination tournaments. The single round robin tournament pattern is exhibited in its preliminary stage group matches while the single elimination tournament pattern is exhibited in the Quarter-final, the Semi-final and the Final stage matches of the tournament.

Round robin tournaments can be constructed using tournament designs. A starter in a cyclic group is selected as the first column and other columns are derived from it cyclically [26]. Possible schedules of play for 4 and 8 teams respectively are presented in Table I.

TABLE I (a)
A TOURNAMENT DESIGN FOR 4 TEAMS

Round I	1-4	3-2
Round II	2-4	1-3
Round III	3-4	2-1

TABLE I (b)
A TOURNAMENT DESIGN FOR 8 TEAMS

Round I	1-8	2-7	3-6	4-5
Round II	2-8	3-1	4-7	5-6
Round III	3-8	4-2	5-1	6-7
Round IV	4-8	5-3	6-2	7-1
Round V	5-8	6-4	7-3	1-2
Round VI	6-8	7-5	1-4	2-3
Round VII	7-8	1-6	2-5	3-4

4 METHODOLOGY

We assumed that none of the qualified 16 teams is a "football minnow" since they all emerged from a strict qualifying series of matches played across the continent. Therefore, the teams should have equal chance of being assigned to any of the groups in the preliminary stage. So, there was no seeding of teams according to status as host or according to previous performance of the teams. We also assumed that all other constraints and factors, as determined by the AFCON tournament

organizers are satisfied.

From the rules governing the 2013 AFCON, sixteen teams are to participate in the tournament. The steps for designing the schedules are discussed as follows:

4.1 Preliminary Stage

Step 1: For the 16 teams, use a simple random sampling without replacement technique, for instance, a random number generator, to select one team at a time. Place the first team selected in group A. Repeat the process placing the second, third and fourth teams selected respectively in Groups B, C and D respectively. Repeat the process until all the teams are allocated to the 4 groups, 4 teams per group. A possible random allocation of the team is presented in Table II. This technique ensures that each possible sample (of equal size) from the population of teams has exactly the same probability of selection [27].

TABLE II
RANDOM ALLOCATION OF THE 16 TEAMS TO GROUPS

Group A	1	5	9	13
Group B	2	6	10	14
Group C	3	7	11	15
Group D	4	8	12	16

Step 2: Use a symmetric Latin square format [28] to schedule the preliminary stage group matches. Recall that a symmetric Latin square is equivalent to a single round robin tournament [29]. For each group, the entry in row i and column j gives the team that opposes i in round j. Each group has $n-1=3$ rounds consisting of $(n/2)=2$ matches each. These are presented in Tables III(a) and (b).

4.2 Single Elimination Stages

The Single Elimination stages comprise the Quarter-final, the Semi-final and the Final stage matches of the tournament.

(i) Quarter and semi-final Stages:

From the rules governing AFCON tournament, two teams with the highest number of points from each group in the preliminary stage proceed to the next stage of the competition which is the quarter-final stage. Then, the winners from the quarter final stage participate in the semi-final stage.

Let *GRPAW1*, *GRPBW3*, *GRPCW5*, and *GRPDW7* represent the winner in each of Groups A, B, C and D respectively. Similarly, let *GRPAR2*, *GRPBR4*, *GRPCR6*, and *GRPDR8* represent the runner-up in each of Groups A, B, C and D respectively. Also, let the winner of each of the 1st, 2nd, 3rd and 4th Quarter final matches be represented by *QF1W*, *QF2W*, *QF3W* and *QF4W* respectively. The schedules are drawn using round I of the tournament designs for 8 and 4 teams respectively. These are presented in Tables IV and V.

(ii) Final Stage:

***Step 3*:** The losers in the semi-final matches play the third place match (losers' final) and the winners play the final match where the overall winner of the competition emerges. This is presented in Table VI.

TABLE III (a)
SYMMETRIC LATIN SQUARES FOR EACH OF THE 4 GROUPS

Group A		**Round**			**Group B**		**Round**			**Group C**		**Round**			**Group D**		**Round**		
		1	**2**	**3**			**1**	**2**	**3**			**1**	**2**	**3**			**1**	**2**	**3**
Teams	**1**	5	9	13	**Teams**	**2**	6	10	14	**Teams**	**3**	7	11	15	**Teams**	**4**	8	12	16
	5	1	13	9		**6**	2	14	10		**7**	3	15	11		**8**	4	16	12
	9	13	1	5		**10**	14	2	6		**11**	15	3	7		**12**	16	4	8
	13	9	5	1		**14**	10	6	2		**15**	11	7	3		**16**	12	8	4

TABLE III (b)

COMPLETE SCHEDULE FOR THE 16 TEAMS FOR PRELIMINARY STAGE GROUP ROUNDS

	GROUP A		GROUP B		GROUP C		GROUP D	
ROUND I	1 vs 5	13 vs 9	2 vs 6	14 vs 10	3 vs 7	15 vs 11	4 vs 8	16 vs 12
ROUND II	1 vs 9	5 vs 13	2 vs 10	6 vs14	3 vs 11	7 vs 15	4 vs 12	8 vs 16
ROUND III	1 vs 13	9 vs 5	2 vs 14	10 vs 6	3 vs 15	11 vs 7	4 vs 16	12 vs 8

TABLE IV
SCHEDULE OF MATCHES FOR THE QUARTER- FINAL STAGE

1st Quarter final match	*GRPAW1* vs *GRPDR8*
2nd Quarter final match	GRPAR2 vs *GRPDW7*
3rd Quarter final match	*GRPBW3* vs *GRPCR6*
4th Quarter final match	*GRPBR4* vs *GRPCW5*

TABLE V
SCHEDULE OF MATCHES FOR THE SEMI-FINAL STAGE

Semi-final match I (SMI)	*QF1W* vs *QF4W*
Semi-final match II(SMII)	*QF2W* vs *QF3W*

TABLE VI
SCHEDULES FOR THE THIRD PLACE AND FINAL MATCHES

Third Place match	SMI Loser vs SMII Loser
Final match	SMI winner vs SMII winner

5. CONCLUSION

In this study, we designed an alternative schedule, which excludes seeding, for the African Cup of Nations (AFCON) Football Tournament. We assumed all other constraints and factors are as arranged by AFCON organizers. The following methods were used:

a. Random allocation of the 16 qualified teams to preliminary groups using simple random sampling without replacement (SRSWOR) technique. This ensures that each possible sample (of size 1) from the population of teams has exactly the same probability of selection.

b. Scheduling of the teams' preliminary stage group matches using the concept of symmetric Latin square design.

c. Scheduling of the quarter-final, semi-final and final stage matches using the Berger tables, a round robin algorithm.

In the schedule designed, the teams have equal probability of being assigned to any of the preliminary groups in the tournament. This is an advantage when compared to the schedule being used by AFCON presently.

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